

## **Fuzzy Time Series Forecasting Model Based on Fuzzy Interval Approach**

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### **Abstract**

For long period of time, different models have been designed and developed. A drawback of traditional forecasting method is that they cannot deal with forecasting problems in which the historical data are linguistic values. To overcome the drawback of the traditional time series, fuzzy time series forecasting is used to forecast. In this paper authors developed and improved fuzzy time series on two factor time variant analysis via considering the latest second factor and previous first factor values and the fuzzy interval. The proposed method uses different approach to define multiset based partitions of the universe of discourse and fuzzy interval approach. Two numerical data sets namely Temperature and cloud density is used to illustrate the proposed method and compare with previous time series models.

**Key words:** Time variant, Multiset, Forecasting, Fuzzy time series, fuzzy interval.

### **1. Introduction**

The classical time series methods can not deal with forecasting problems in which the values of time series are linguistic terms represented by fuzzy sets. Forecasting using fuzzy time series has been widely used in many activities. It arises in forecasting the weather, earthquakes, stock fluctuations and any phenomenon indexed by variables that change unpredictably in time. Therefore in 1993, Song and Chissom proposed the concepts of fuzzy time series and outlined equations and approximate reasoning based on the fuzzy set theory introduced by Zadeh (1965). They also presented the models on time – variant and time invariant fuzzy time series [15], [16], [17] to deal with forecasting problems in which the historical data is represented by linguistic values. They asserted that all traditional forecasting methods fail when the historical enrollment data are composed of linguistic values. Sullivan and Woodall (1994) described a Markov model using linguistic values directly but with membership function of the fuzzy approach replaced by analogues probability function. Instead of complicated maximum – minimum composition operations Chen (1996) used a simple arithmetic operation for time series forecasting. Hwang, Chen and Lee (1998) presented a method of forecasting enrollments using fuzzy time series based on the concept that the variation of enrollment of this year is related to the trend of the enrollments of the past years. Huarng (2001) pointed out that the length of intervals will affect the forecasting accuracy rate and a proper choice of length of intervals can greatly improve the forecasting result. He presented the distribution based length approach and the average based length approach to deal with forecasting problems based on the intervals with different lengths. In recent years researchers focused on the research topic of using fuzzy time series for handling forecasting problems. A number of works have been reported on order one, high order, Heuristic high order, single factor, two factor, multi factor, dual factor and integrated models. The formulation of fuzzy relation is one of the key issues affecting forecasting results. Li and Cheng (2010) proposed a forecasting model based on the hidden Markov model by enhancing Sullivan and Woodall's (1994) work to allow handling of two factor forecasting problems. Huang et al (2011) developed a forecasting model based on two computational methods fuzzy

time series and particle swarm optimization. In most of the realistic situation, we have to deal with collection of objects in which repetition of elements is significant. In this situation bags (multisets) are very useful structures. Multiset theory (Bag) was introduced by Cerf et al.(1971). Peterson (1976) and Yager (1986) made contributions to it. Further study was carried out by Jena et al. (2001).

In this paper, we propose an efficient fuzzy time series forecasting model based on the concept of characterization of Bag to form the intervals of different length and fuzzy intervals to deal with the temperature prediction. The remainder of this paper is organized as follows. In section 2, the basic concept of fuzzy time series is briefly introduced. In section 3, the new forecasting model based on Fuzzy interval and multiset is proposed. Section 4 presents a performance evaluation of the model and a comparison of the results. The conclusions are discussed in section 5.

## 2. Fuzzy time series

In the following, we briefly review some basic concepts of fuzzy time series and its forecasting frame work.

**Definition 1:** A *fuzzy set*  $A$  is defined as an uncertain subset of the fundamental set  $X$ .

$$A = \{x, \mu_A(x) | x \in X\}$$

The uncertainty is assessed by the membership function  $\mu_A(x)$ .

**Definition 2:** A *normalized membership function*  $\mu_A(x)$  is defined as follows:

$$0 \leq \mu_A(x) \leq 1 \quad \forall x \in \mathbb{R}$$

$$\exists x_l, x_r \text{ with } \mu_A(x) = 1 \quad \forall x \in [x_l, x_r].$$

**Definition 3:** A fuzzy set is referred to as *convex* if its membership function  $\mu_A(x)$  monotonically decreases on each side of the maximum value, ie, if

$$\mu_A(x_2) \geq \min [\mu_A(x_1); \mu_A(x_3)] \quad \forall x_1, x_2, x_3 \in \mathbb{R} \text{ with } x_1 \leq x_2 \leq x_3 \text{ applies.}$$

**Definition 4: [2]** A convex fuzzy set is referred to as a *fuzzy number* if its membership function is piecewise continuous and if it has the functional value  $\mu_A(x) = 1$  at precisely one of the  $x$  values with  $x = x_l = x_r$  according to the following equation

$$x_l = \min [x \in \mathbb{R} | \mu_A(x) = 1] \text{ and}$$

$$x_r = \max [x \in \mathbb{R} | \mu_A(x) = 1]$$

In the case  $x_l < x_r$  the fuzzy set constitutes a *fuzzy interval*.

The definition of fuzzy time series used in this paper was first proposed by Song and Chissom [15, 16, 17].

**Definition 5:** Let  $Y(t) \{t = 0, 1, 2, 3, \dots\}$ , a subset of  $\mathbb{R}$ , be the universe of discourse on which fuzzy sets  $f_i(t)$  ( $i = 1, 2, 3, \dots$ ) are defined and let  $F(t)$  be the collection of  $f_i(t)$ . Then  $F(t)$  is defined as *fuzzy time series* on  $Y(t)$ .

From this definition we can see that , (1)  $F(t)$  is the function of time

(2)  $F(t)$  can be regarded as a *linguistic variable*, which is a variable whose values are linguistic values represented by fuzzy sets.

(3)  $f_i(t)$  ( $i = 1,2,3,\dots$ ) are possible linguistic values of  $F(t)$ , where  $f_i(t)$  ( $i = 1,2,3,\dots$ ) are represented by fuzzy sets.

Song and Chissom employed a fuzzy relational equation to develop their forecasting model under the assumption that the observations at time  $t$  are dependent only upon the accumulated results of the observation at previous times, which is defined as follows.

**Definition 6:** Suppose  $F(t)$  is caused only by  $F(t-1)$  and is denoted by  $F(t-1) \rightarrow F(t)$ , then there is a fuzzy relationship between  $F(t)$  and  $F(t-1)$  and can be expressed as the fuzzy relational equation  $F(t) = F(t-1) \circ R(t,t-1)$ . Here ' $\circ$ ' is max-min composition operator. The relation  $R$  is called first-order model of  $F(t)$ .

Further, if fuzzy relation  $R(t,t-1)$  of  $F(t)$  is independent of time  $t$ , that is to say, for different times  $t_1$  and  $t_2$ ,  $R(t_1,t_1-1) = R(t_2,t_2-1)$ , then  $F(t)$  is called a *time invariant* fuzzy time series otherwise  $F(t)$  is *time variant*.

**Definition 7:** Suppose  $F(t-1) = A_i$  and  $F(t) = A_j$  a *fuzzy logical relationship* can be defined as  $A_i \rightarrow A_j$  where  $A_i$  and  $A_j$  are called the left hand side and the right hand side of the fuzzy logical relationship respectively.

**Definition 8:** If  $F(t)$  is caused by more fuzzy sets  $F(t-n), F(t-n+1), \dots, F(t-1)$  the fuzzy relationship is represented by  $A_{i1}, A_{i2}, A_{i3}, \dots, A_{in} \rightarrow A_j$ , where  $F(t-n) = A_{i1}, F(t-n+1) = A_{i2}, \dots, F(t-1) = A_{in}$ . This relationship is called  $n^{\text{th}}$  order fuzzy time series model.

**Definition 9:** Suppose  $F(t)$  is caused by  $F(t-1), F(t-2), \dots, F(t-m)$ , ( $m > 0$ ) simultaneously and the relations are time variant. Then  $F(t)$  is said to be time variant fuzzy time series and the relation can be expressed as the fuzzy relational equation  $F(t) = F(t-1) \circ R^w(t, t-1)$ , here  $w > 1$  is a time (number of years) parameter by which the forecast  $F(t)$  is being affected, various complicated computational methods are available for the computations of the relation  $R^w(t, t-1)$ .

**Definition 10:** [7] A collection of elements which may contain duplicates is called a *Multiset (bag)*. Formally if  $X$  is a set of elements, a bag drawn from the set  $X$  is represented by a function count  $B$  or  $C_B$  defined as  $C_B: X \rightarrow N$ , where  $N$  represents the set of non-negative integers. For each  $x \in X$ ,  $C_B(x)$  is a *characteristic value* of  $x$  in  $B$  and indicates the number of occurrences of the elements  $x$  in  $B$ .

### 3. Proposed Model

In this section we introduce a model to forecast the Temperature of Taipei. The historical data of daily average Temperature and the daily Cloud density of Taipei from June 1996 to September 1996 are considered from [10]. The step-wise procedure of the proposed model of fuzzy time series is detailed as follows.

#### Step 1:

Define the universe of discourse  $U = [\text{low}, \text{up}]$ , which can cover all observations of Temperature in the selected month of historical data set. Initially partition the universe of discourse into seven linguistic intervals,  $U_i$ , of equal length. Further divide the intervals using Multiset Characterization denoted by  $v_1, v_2, \dots, v_n$ .

#### Step 2:

Define and partition the corresponding month of universe of discourse of cloud density equal to the number of intervals of Temperature.

**Step 3:**

Assign weight to the intervals as per the division. For example if there is no division assign  $w_1 = 1$ , and if there is an interval divided by 2 then assign to each interval as  $w_2 = \frac{1}{2}$  and  $w_3 = \frac{1}{2}$  etc.

**Step 4:**

Assume that there are n linguistic terms (i.e.  $A_1, A_2, \dots, A_n$ ) described by fuzzy sets which is defined on the universal set U by the following formula:

$$\mu_{A_i}(v_i) = \frac{1}{1+[q(v_m - v'_m)]^2}$$

where  $v_m$  represents the actual value of the interval,  $v'_m$  is the mid value of the corresponding interval and q (a constant) is chosen in such a way that it ensures conversion of definite quantitative values into fuzzy values.

$$A_i = \left( \mu_{A_i}(v_i) / v_i \right), v_i \in U, \mu_{A_i}(v_i) \in [0,1] \text{ is a fuzzy set.}$$

The fuzzification of Temperature data are denoted by  $A^1, A^2, \dots$  corresponding to first day, second day etc.

**Step 5:**

As in step 1 through step 4 fuzzify the values of Cloud density of the corresponding month given in the historical data set. Name the fuzzy sets as  $B^i$ .

**Step 6:**

Consider the previous fuzzy set of Temperature  $A^{i-1}$  and the current fuzzy set of Cloud density  $B^i$ . Compare them with respect to the membership values as follows:

$$\text{Max } [A^{i-1}, B^i]$$

The new set of membership values will be considered for  $A^i$ .

**Step 7:**

Defuzzify the fuzzified forecasted values of the first factor (temperature) of fuzzy time series obtained in step 6. By the following Rules defuzzify the fuzzified forecasted values.

**Rule 1:**

If there is only one 1 occurs among the memberships of the fuzzy set  $A^i$ , then consider the corresponding interval  $[x_l, x_r]$  where 1 lies. Let the actual value of the corresponding period be x. Calculate

$$\min [|x_l - x|, |x_m - x|, |x_r - x|]$$

where  $x_l, x_r$  denote the left and right end values of the interval and  $x_m$  is the mid value of the interval. If the minimum value occurs at  $|x_r - x|$ , then the forecasting value of  $A^i$  is  $\frac{w_i x_r + w_j x}{w_i + w_j}$  where  $w_i$  is the weight of the interval where  $x_r$  lies and  $w_j$  is the weight of the interval where x lies.

**Rule 2:**

If more than 1 occurs consecutively among the memberships in a fuzzy set  $A^i$ , then form the fuzzy interval  $[x_l, x_r]$  where  $x_l$  is the midvalue of the interval where the first 1 occurs and  $x_r$  is the midvalue of the interval where the last 1 occurs. Then the forecasting value of  $A^i$  is as in Rule 1.

**Rule 3:**

If more than 1 occurs among the memberships in a fuzzy set  $A^i$  (not consecutively) in different intervals then consider the interval for which the maximum weight exist. Then follow Rule 1.

**Rule 4:**

Consider the case where the existence of only one 1 occurs among the memberships and some consecutive 1<sup>s</sup> exist in some other places in a fuzzy set  $A^i$ . Form the fuzzy interval as in rule 2 for the consecutive 1<sup>s</sup>. Sum the weights of the interval where the consecutive 1<sup>s</sup> occur.

$w_m = \text{Max} [\text{weight of the interval in which only one 1 occur, Sum the weights of the interval where the consecutive 1<sup>s</sup> occur}].$

Choose the interval corresponding to  $w_m$ . Then follow Rule 1.

**Rule 5:**

Suppose two fuzzy intervals can happen among the memberships in a fuzzy set  $A^i$ . Find the maximum of the sum of the weights of the intervals corresponding to first and second fuzzy intervals. Choose the intervals corresponding to the maximum weight. Then follow Rule 1.

**Step 8:**

Choose  $\beta$  in (0, 1). Make an error analysis as follows:

$$\text{New forecasted value} = \frac{\text{Actual value} + \text{forecasted value} - 2\beta}{2}$$

Compute Root Mean Square Error (RMSE) values for different  $\beta$  values on new forecasted values. Fix  $\beta$  corresponding to minimum RMSE value.

The forecasted values with respect to this  $\beta$  are the expected forecasted values.

Table 1: Intervals by multiset, midpoints of average temperature of June 1996

Interval	Mid point	Interval	Mid point
$v_1 = [26, 26.71)$	$m_1 = 26.355$	$v_{10} = [28.941, 29.043)$	$m_{10} = 28.992$
$v_2 = [26.71, 27.42)$	$m_2 = 27.065$	$v_{11} = [29.043, 29.144)$	$m_{11} = 29.094$
$v_3 = [27.42, 27.775)$	$m_3 = 27.598$	$v_{12} = [29.144, 29.246)$	$m_{12} = 29.195$
$v_4 = [27.775, 28.13)$	$m_4 = 27.953$	$v_{13} = [29.246, 29.347)$	$m_{13} = 29.296$
$v_5 = [28.13, 28.308)$	$m_5 = 28.219$	$v_{14} = [29.347, 29.449)$	$m_{14} = 29.398$
$v_6 = [28.31, 28.485)$	$m_6 = 28.396$	$v_{15} = [29.449, 29.55)$	$m_{15} = 29.499$
$v_7 = [28.485, 28.66)$	$m_7 = 28.574$	$v_{16} = [29.55, 29.905)$	$m_{16} = 29.728$
$v_8 = [28.663, 28.84)$	$m_8 = 28.751$	$v_{17} = [29.905, 30.26)$	$m_{17} = 30.083$
$v_9 = [28.84, 28.941)$	$m_9 = 28.891$	$v_{18} = [30.26, 31)$	$m_{18} = 30.63$

26.1	A <sup>1</sup>	1	.99	.98	.97	.96	.95	.94	.93	.93	.92	.92	.91	.91	.90	.90	.88	.86	.83
27.6	A <sup>2</sup>	.98	1	1	1	1	.99	.99	.99	.98	.98	.98	.98	.97	.97	.97	.96	.94	.92
29.0	A <sup>3</sup>	.93	.96	.98	.99	.99	1	1	1	1	1	1	1	1	1	.99	.99	.97	
30.5	A <sup>4</sup>	.89	.89	.92	.94	.95	.96	.96	.97	.97	.98	.98	.98	.99	.99	.99	.99	1	1
30.0	A <sup>5</sup>	.88	.92	.95	.96	.97	.97	.98	.98	.99	.99	.99	.99	1	1	1	1	1	1
29.5	A <sup>6</sup>	.91	.94	.97	.98	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	.99
29.7	A <sup>7</sup>	.90	.94	.96	.97	.98	.98	.99	.99	.99	1	1	1	1	1	1	1	1	.99
29.4	A <sup>8</sup>	.92	.95	.97	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	1	.99
28.8	A <sup>9</sup>	.94	.97	.99	.99	1	1	1	1	1	1	1	1	1	1	1	.99	.98	.97
29.4	A <sup>10</sup>	.92	.95	.97	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	1	.99
29.3	A <sup>11</sup>	.92	.95	.97	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	.99	.98
28.5	A <sup>12</sup>	.96	.98	.99	1	1	1	1	1	1	1	1	.99	.99	.99	.99	.98	.96	
28.7	A <sup>13</sup>	.95	.97	.99	.99	1	1	1	1	1	1	1	1	1	.99	.99	.98	.96	
27.5	A <sup>14</sup>	.99	1	1	1	.99	.99	.99	.98	.98	.98	.97	.97	.97	.96	.95	.94	.91	
29.5	A <sup>15</sup>	.91	.94	.97	.98	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	.99
28.8	A <sup>16</sup>	.94	.97	.99	.99	1	1	1	1	1	1	1	1	1	1	1	.99	.98	.97
29.0	A <sup>17</sup>	.93	.96	.98	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.97
30.3	A <sup>18</sup>	.87	.91	.93	.95	.96	.97	.97	.98	.98	.98	.99	.99	.99	.99	.99	1	1	1
30.2	A <sup>19</sup>	.87	.91	.94	.95	.96	.97	.97	.98	.98	.99	.99	.99	.99	.99	1	1	1	1
30.9	A <sup>20</sup>	.83	.87	.90	.92	.93	.94	.95	.96	.96	.96	.97	.97	.97	.97	.98	.98	.99	.99
30.8	A <sup>21</sup>	.84	.88	.91	.92	.94	.95	.95	.96	.96	.97	.97	.97	.97	.98	.98	.98	.99	1
28.7	A <sup>22</sup>	.95	.97	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.98	.96
27.8	A <sup>23</sup>	.98	.99	1	1	1	1	.99	.99	.99	.99	.98	.98	.98	.98	.97	.96	.95	.93
27.4	A <sup>24</sup>	.99	1	1	1	.99	.99	.99	.98	.98	.98	.97	.97	.97	.96	.96	.95	.93	.91
27.7	A <sup>25</sup>	.98	1	1	1	1	1	.99	.99	.99	.98	.98	.98	.98	.97	.97	.96	.95	.92
27.1	A <sup>26</sup>	.99	1	1	.99	.99	.98	.98	.97	.97	.97	.96	.96	.95	.95	.95	.94	.92	.89
28.4	A <sup>27</sup>	.96	.98	.99	1	1	1	1	1	1	1	1	.99	.99	.99	.99	.98	.97	.95
27.8	A <sup>28</sup>	.98	.99	1	1	1	1	.99	.99	.99	.99	.98	.98	.98	.98	.97	.96	.95	.93
29.0	A <sup>29</sup>	.93	.96	.98	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.97
30.2	A <sup>30</sup>	.87	.91	.94	.95	.96	.97	.97	.98	.98	.99	.99	.99	.99	.99	1	1	1	1

Table 2 :Fuzzification of daily average Temperature of June 1996.

Table 3: Intervals and midpoints corresponding to cloud density of June 1996.

Interval	Mid point	Interval	Mid point
k <sub>1</sub> = [10, 14.78)	m <sub>1</sub> = 12.39	k <sub>10</sub> = [53.02, 57.8)	m <sub>10</sub> = 55.41
k <sub>2</sub> = [14.78, 19.56)	m <sub>2</sub> = 17.17	k <sub>11</sub> = [57.8, 62.58)	m <sub>11</sub> = 60.91
k <sub>3</sub> = [19.56, 24.34)	m <sub>3</sub> = 21.95	k <sub>12</sub> = [62.58, 67.36)	m <sub>12</sub> = 64.97
k <sub>4</sub> = [24.34, 29.12)	m <sub>4</sub> = 26.73	k <sub>13</sub> = [67.36, 72.14)	m <sub>13</sub> = 69.75
k <sub>5</sub> = [29.12, 33.9)	m <sub>5</sub> = 31.51	k <sub>14</sub> = [72.14, 76.92)	m <sub>14</sub> = 74.53
k <sub>6</sub> = [33.9, 38.68)	m <sub>6</sub> = 36.29	k <sub>15</sub> = [76.92, 81.7)	m <sub>15</sub> = 79.31
k <sub>7</sub> = [38.68, 43.46)	m <sub>7</sub> = 41.07	k <sub>16</sub> = [81.7, 86.48)	m <sub>16</sub> = 84.09
k <sub>8</sub> = [43.46, 48.24)	m <sub>8</sub> = 45.85	k <sub>17</sub> = [86.48, 91.26)	m <sub>17</sub> = 88.87
k <sub>9</sub> = [48.24, 53.02)	m <sub>9</sub> = 50.63	k <sub>18</sub> = [91.26, 96)	m <sub>18</sub> = 93.63

36	B <sup>1</sup>	.15	.22	.34	.54	.83	.93	.80	.51	.32	.21	.15	.11	.08	.06	.05	.04	.03	.03
23	B <sup>2</sup>	.47	.75	.99	.88	.58	.36	.23	.16	.12	.09	.07	.05	.04	.04	.03	.03	.02	.02
23	B <sup>3</sup>	.47	.75	.99	.88	.58	.36	.23	.16	.12	.09	.07	.05	.04	.04	.03	.03	.02	.02
10	B <sup>4</sup>	.95	.66	.41	.26	.18	.13	.09	.07	.06	.05	.04	.03	.03	.02	.02	.02	.02	.01
13	B <sup>5</sup>	1	.85	.56	.35	.23	.16	.11	.08	.07	.05	.04	.04	.03	.03	.02	.02	.02	.02
30	B <sup>6</sup>	.24	.38	.61	.90	.98	.72	.45	.28	.19	.13	.10	.08	.06	.05	.04	.03	.03	.02
45	B <sup>7</sup>	.09	.11	.16	.23	.35	.57	.87	.99	.76	.48	.30	.20	.14	.10	.08	.06	.05	.04
35	B <sup>8</sup>	.16	.24	.37	.59	.89	.98	.73	.46	.29	.19	.14	.10	.08	.06	.05	.04	.03	.03
26	B <sup>9</sup>	.35	.56	.86	.99	.77	.49	.31	.20	.14	.10	.08	.06	.05	.04	.03	.03	.02	.02
21	B <sup>10</sup>	.57	.87	.99	.75	.48	.30	.20	.14	.10	.08	.06	.05	.04	.03	.03	.02	.02	.02
43	B <sup>11</sup>	.10	.13	.18	.27	.43	.69	.96	.92	.63	.39	.25	.17	.12	.09	.07	.06	.05	.04
40	B <sup>12</sup>	.12	.16	.23	.36	.58	.88	.91	.75	.47	.30	.20	.14	.10	.08	.06	.05	.04	.03
30	B <sup>13</sup>	.24	.38	.61	.90	.98	.72	.45	.28	.19	.13	.10	.08	.06	.05	.04	.03	.03	.02
29	B <sup>14</sup>	.27	.42	.67	.95	.94	.65	.41	.26	.18	.13	.09	.07	.06	.05	.04	.03	.03	.02
30	B <sup>15</sup>	.24	.38	.61	.90	.98	.72	.45	.28	.19	.13	.10	.08	.06	.05	.04	.03	.03	.02
46	B <sup>16</sup>	.08	.11	.15	.21	.32	.51	.80	1	.82	.53	.33	.22	.15	.11	.08	.06	.05	.04
55	B <sup>17</sup>	.05	.07	.08	.11	.15	.22	.34	.54	.84	1	.79	.50	.31	.21	.14	.11	.08	.06
19	B <sup>18</sup>	.70	.97	.92	.63	.39	.25	.17	.12	.09	.07	.06	.05	.04	.03	.03	.02	.02	.02
15	B <sup>19</sup>	.94	.96	.67	.42	.27	.18	.13	.10	.07	.06	.05	.04	.03	.03	.02	.02	.02	.02
56	B <sup>20</sup>	.05	.06	.08	.10	.14	.20	.31	.49	.78	1	.85	.55	.35	.23	.16	.11	.08	.07
60	B <sup>21</sup>	.04	.05	.06	.08	.11	.15	.22	.33	.53	.83	1	.80	.51	.32	.21	.15	.11	.08
96	B <sup>22</sup>	.01	.02	.02	.02	.02	.03	.03	.04	.05	.06	.07	.09	.13	.18	.26	.41	.66	.95
63	B <sup>23</sup>	.04	.05	.06	.07	.09	.12	.17	.25	.40	.63	.93	.96	.69	.43	.27	.18	.13	.10
28	B <sup>24</sup>	.29	.46	.73	.98	.89	.59	.37	.24	.16	.12	.09	.07	.05	.04	.24	.03	.03	.02
14	B <sup>25</sup>	.97	.91	.61	.38	.25	.17	.12	.09	.07	.06	.04	.04	.03	.03	.02	.02	.02	.02
25	B <sup>26</sup>	.39	.62	.91	.97	.70	.44	.28	.19	.13	.10	.07	.06	.05	.04	.03	.03	.03	.02
29	B <sup>27</sup>	.27	.42	.67	.95	.94	.65	.41	.26	.18	.13	.09	.07	.06	.05	.04	.03	.03	.02
55	B <sup>28</sup>	.05	.07	.08	.11	.15	.22	.39	.54	.84	1	.79	.50	.31	.21	.14	.11	.08	.06
29	B <sup>29</sup>	.27	.42	.67	.95	.94	.65	.41	.26	.18	.13	.09	.07	.06	.05	.04	.03	.03	.02
19	B <sup>30</sup>	.70	.97	.92	.63	.39	.25	.17	.12	.09	.07	.06	.05	.04	.03	.03	.02	.02	.02

Table 4: Fuzzification of Daily cloud density of June 1996

Actual data	Fuzzy Interval	Forecast value	Actual data	Fuzzy Interval	Forecast value
26.1	-	-	28.8	[28.75125, 30.0825]	28.74
27.6	[26, 26.71]	27.4	29.0	[28.219, 29.499]	29.16
29.0	[27.065, 28.219]	28.72	30.3	[28.396, 29.499]	30.27
30.5	[28.396, 29.499]	30.41	30.2	[29.728, 30.63]	30.16
30.0	[30.083, 30.63]	30.02	30.9	[29.499, 30.63]	30.87
29.5	[29.296, 30.63]	29.46	30.8	[30.26, 31]	30.71
29.7	[28.891, 30.082]	29.68	28.7	[30.26, 31]	29.28
29.4	[28.992, 30.082]	29.41	27.8	[28.219, 29.398]	27.84
28.8	[28.751, 30.083]	28.79	27.4	[27.598, 28.396]	27.49
29.4	[28.219, 29.499]	29.41	27.7	[27.065, 27.953]	27.61
29.3	[28.751, 30.083]	29.32	27.1	[27.065, 28.396]	27.19
28.5	[28.751, 29.728]	28.54	28.4	[27.065, 27.598]	28.14
28.7	[27.953, 29.195]	28.67	27.8	[27.953, 29.094]	27.93
27.5	[28.219, 29.398]	27.64	29.0	[27.598, 28.396]	28.84
29.5	[27.065, 27.9525]	28.87	30.2	[28.39625, 29.49929]	30.1

Table 6: Formation of fuzzy intervals and forecasting values for June 1996 temperature

Table 5: Values of max  $[A^{i-1}, B^i]$

27.6	A <sup>2</sup>	1	.99	.99	.97	.96	.95	.94	.93	.93	.92	.92	.91	.91	.90	.90	.88	.86	.83
29.0	A <sup>3</sup>	.98	1	1	1	1	.99	.99	.99	.98	.98	.98	.98	.97	.97	.97	.96	.94	.92
30.5	A <sup>4</sup>	.95	.96	.98	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.97
30.0	A <sup>5</sup>	1	.89	.92	.94	.95	.96	.96	.97	.97	.98	.98	.98	.99	.99	.99	.99	1	1
29.5	A <sup>6</sup>	.88	.92	.95	.96	.98	.97	.98	.98	.99	.99	.99	.99	1	1	1	1	1	1
29.7	A <sup>7</sup>	.91	.94	.97	.98	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	.99
29.4	A <sup>8</sup>	.90	.94	.96	.97	.98	.98	.99	.99	.99	1	1	1	1	1	1	1	1	.99
28.8	A <sup>9</sup>	.92	.95	.97	.99	.99	.99	.99	1	1	1	1	1	1	1	1	1	1	.99
29.4	A <sup>10</sup>	.94	.97	.99	.99	1	1	1	1	1	1	1	1	1	1	1	.99	.98	.97
29.3	A <sup>11</sup>	.92	.95	.97	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	1	.99
28.5	A <sup>12</sup>	.92	.95	.97	.98	.99	.99	.99	1	1	1	1	1	1	1	1	1	.99	.98
28.7	A <sup>13</sup>	.96	.98	.99	1	1	1	1	1	1	1	1	1	.99	.99	.99	.99	.98	.96
27.5	A <sup>14</sup>	.95	.97	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.98	.96
29.5	A <sup>15</sup>	.99	1	1	1	.99	.99	.99	.98	.98	.98	.98	.97	.97	.97	.96	.95	.94	.91
28.8	A <sup>16</sup>	.91	.94	.97	.98	.98	.99	.99	1	1	1	1	1	1	1	1	1	1	.99
29.0	A <sup>17</sup>	.94	.97	.99	.99	1	1	1	1	1	1	1	1	1	1	1	.99	.98	.97
30.3	A <sup>18</sup>	.93	.97	.98	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.97
30.2	A <sup>19</sup>	.87	.96	.93	.95	.96	.97	.97	.98	.98	.98	.99	.99	.99	.99	.99	1	1	1
30.9	A <sup>20</sup>	.87	.91	.94	.95	.96	.97	.97	.98	.98	1	.99	.99	.99	.99	1	1	1	1
30.8	A <sup>21</sup>	.83	.87	.90	.92	.93	.94	.95	.96	.96	.96	1	.97	.97	.98	.98	.99	.99	1
28.7	A <sup>22</sup>	.84	.88	.91	.92	.94	.95	.95	.96	.96	.97	.97	.97	.98	.98	.98	.99	.99	1
27.8	A <sup>23</sup>	.95	.97	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.98	.96
27.4	A <sup>24</sup>	.98	.99	1	1	1	1	.99	.99	.99	.99	.98	.98	.98	.98	.97	.96	.95	.93
27.7	A <sup>25</sup>	.99	1	1	1	.99	.99	.99	.98	.98	.98	.97	.97	.97	.96	.96	.95	.93	.91
27.1	A <sup>26</sup>	.98	1	1	1	1	1	.99	.99	.99	.99	.98	.98	.98	.98	.97	.96	.95	.92
28.4	A <sup>27</sup>	.99	1	1	.99	.99	.98	.98	.97	.97	.97	.96	.96	.95	.95	.95	.94	.92	.89
27.8	A <sup>28</sup>	.96	.98	.99	1	1	1	1	1	1	1	1	.99	.99	.99	.99	.98	.97	.95
29.0	A <sup>29</sup>	.98	.99	1	1	1	1	.99	.99	.99	.99	.98	.98	.98	.98	.97	.96	.95	.93
30.2	A <sup>30</sup>	.93	.97	.98	.99	.99	1	1	1	1	1	1	1	1	1	1	.99	.99	.97

Table 7: Comparison of forecasting values with the existing values

Month	Lee et.al(2006)	Lee et.al(2007)	Wang and Chen(2009)	Proposed Method
June	1.44	1.24	0.53	0.41
July	1.59	1.23	0.71	0.40
Aug	1.26	1.09	0.32	0.54
Sept	1.89	1.28	0.74	0.61

**Step 9:**

Calculate the average forecasting error rates (AFER) as follows:



$AFER = \frac{\sum_{i=1}^n |(F_i - A_i)/A_i|}{n} \times 100\%$  where  $A_i$  denotes the actual value of day  $i$  and  $F_i$  denotes the forecasting value of day  $i$  respectively.

**Step 10:**

Comparing the AFER with some existing results .

**4. Performance evaluation of the model**

Consider the daily average temperature of June 1996. The universe of discourse is  $U = [26, 31]$ . Partition the universe of discourse into seven intervals as follows:

$u_1 = [26, 26.71)$ ,  $u_2 = [26.71, 27.42)$ ,  $u_3 = [27.42, 28.13)$ ,  $u_4 = [28.13, 28.84)$ ,  $u_5 = [28.84, 29.55)$ ,  $u_6 = [29.55, 30.26)$  and  $u_7 = [30.26, 31)$ . By step 1 and step 3 we have the values as in Table 1. Fuzzify the values of daily average temperature of June 1996 as in step 4 and the values are given in Table 2. Table 3 represents the intervals & mid points corresponding to the cloud density of June 1996 by step 2. By step5, fuzzify the data of cloud density of June 1996 and are given in Table 4. Table 5 gives the values of  $\text{Max} [A^{i-1}, B^i]$  by step 6. By rules given in step 7, the fuzzy intervals are formed and by step 8, the forecasted values for  $\beta = 0.025$  corresponding to the daily average temperature of June 1996 are given in Table 6.

**5. Conclusions**

In this paper, we have presented a new efficient method for forecasting the daily average temperature based on the fuzzy intervals. An algorithm is given to form the fuzzy intervals. From the experimental results, we infer the proposed method gets higher forecasting accuracy rates than the methods presented in (Lee et.al., 2006; Lee et.al., 2007; Wang and Chen., 2009).

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